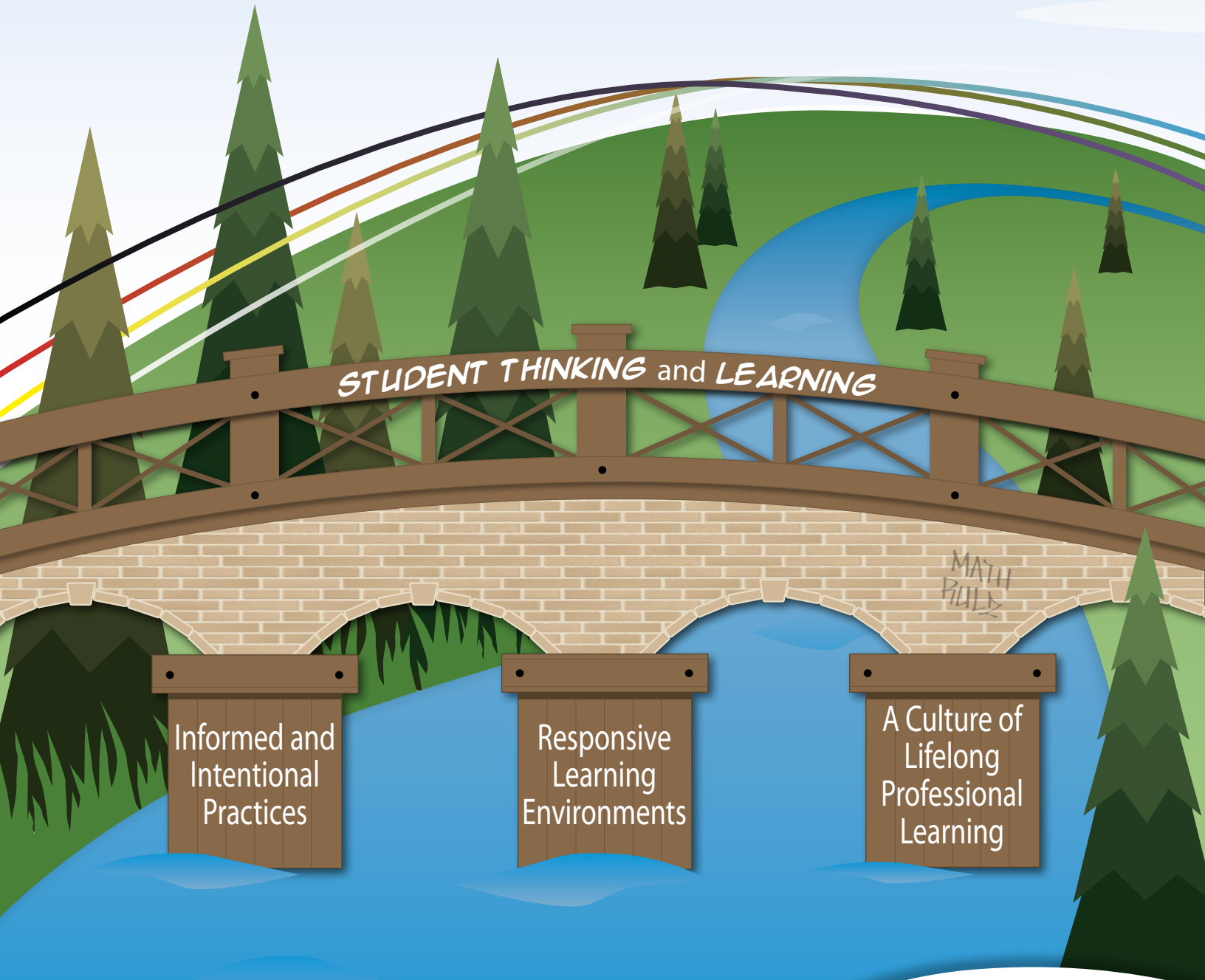
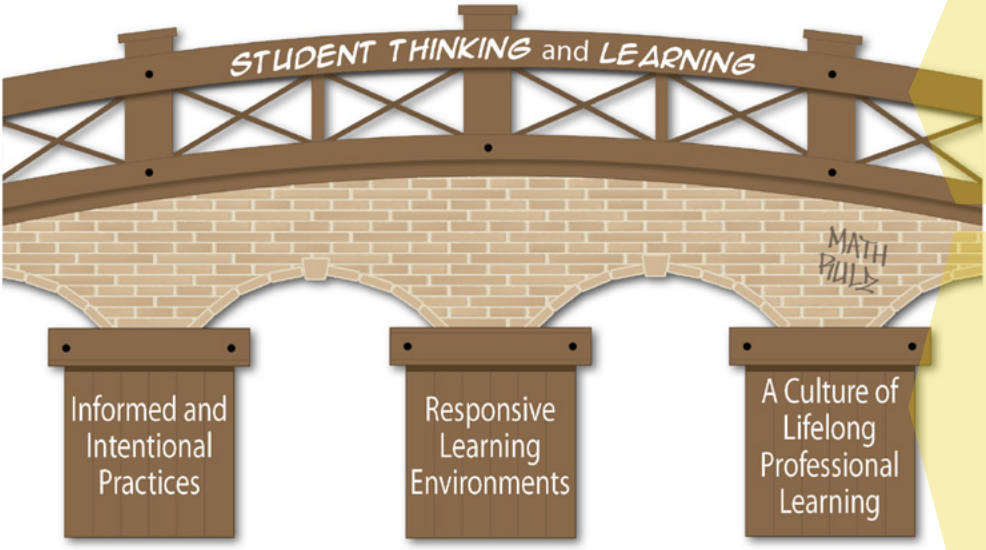
PILLARS *for* Teaching *and* Learning MATHEMATICS



Manitoba

Overview

Pillars for Teaching and Learning Mathematics is a framework that supports improvement and sustained growth in Kindergarten to Grade 12 mathematics education in Manitoba. This resource was initiated by Manitoba Education and Training and developed in collaboration with members of the provincial Numeracy Leaders' Network. The framework provides educators with a starting point for reflection, discussion, and learning. It will set direction for professional learning and help reinforce consistent language and practices for the teaching and learning of mathematics in Manitoba.



The image of a bridge is a reminder to work together to connect research and powerful practices to enhance mathematics learning for all students. It represents the connections between all educators in northern, rural, and urban communities throughout the province.

The bridge is supported by three **interconnected pillars** for teaching and learning mathematics, all of which are necessary to support **student thinking and learning**. They represent the need to foster

- informed and intentional practices
- responsive learning environments
- a culture of lifelong professional learning

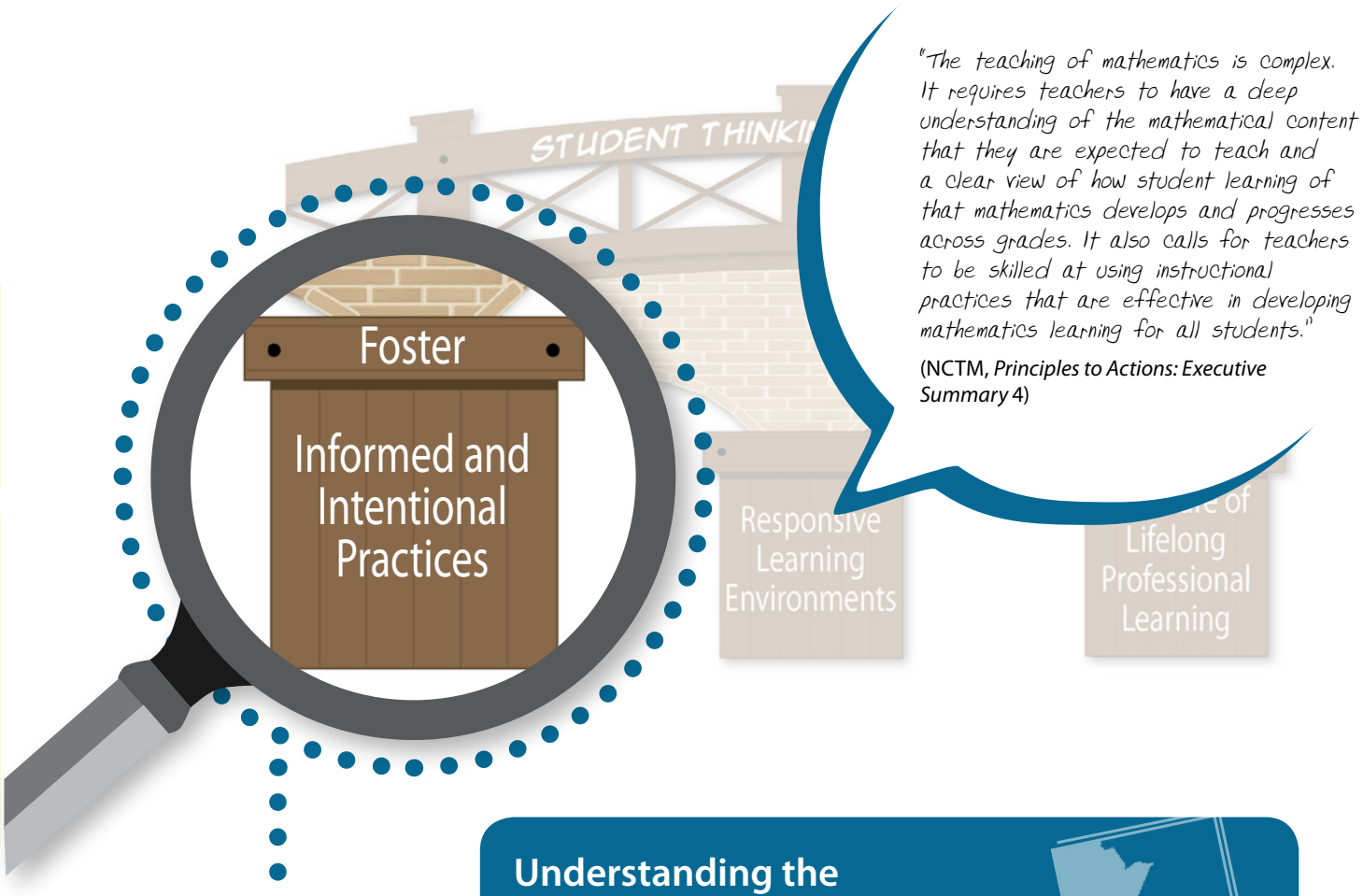
These three pillars guide, motivate, and inspire educators (teachers, school leaders, school division leaders, educational assistants, and parents/guardians) in all aspects of quality teaching and learning of mathematics, including

- classroom planning, instruction, and assessment
- professional learning
- school division and school planning

The pillars and their respective components are described separately in the framework; however, they are connected concepts that work together to support student thinking and learning. This framework is intended to help educators foster the ideas inherent in the pillars, with the end goal of improving student learning.

> "We must move from 'pockets of excellence' to 'systemic excellence' by providing mathematics education that supports the learning of all students at the highest possible level."
>
> (National Council of Teachers of Mathematics [NCTM], *Principles to Actions* 3)

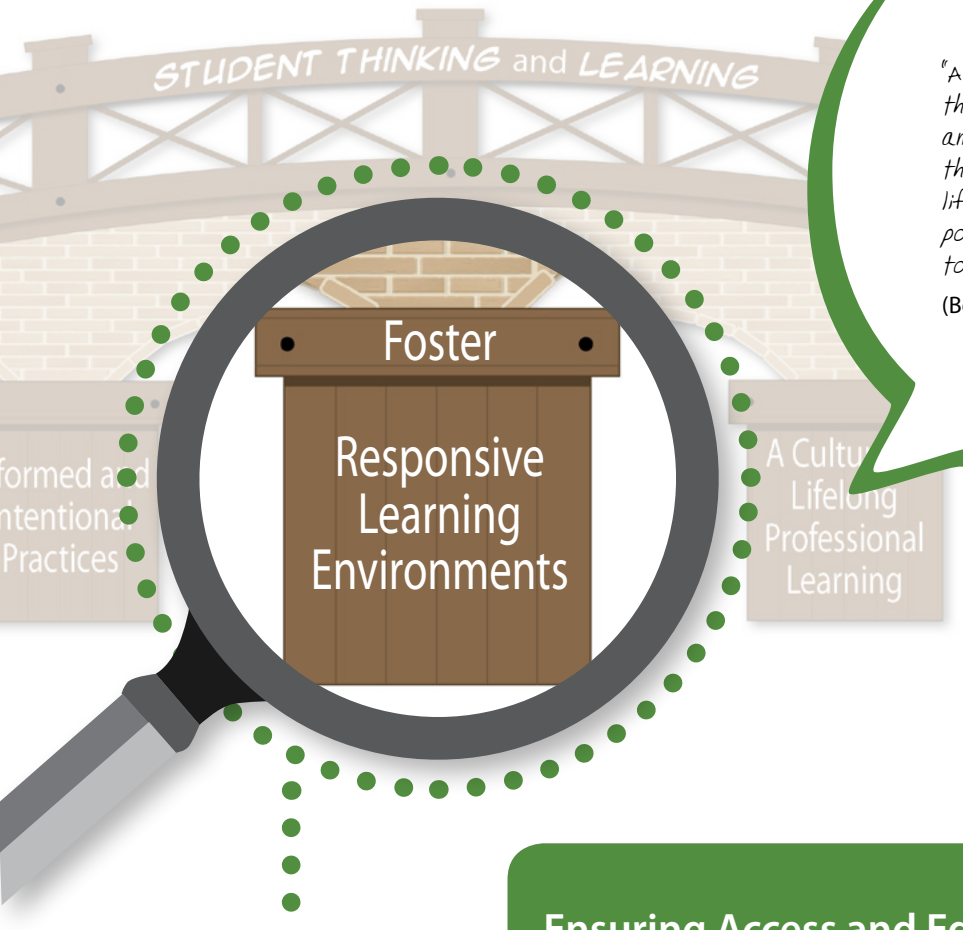
The framework will inspire the collaborative creation and sharing of **pictures of practice (POP)** that have an impact on and exemplify quality teaching practices and professional learning. The pictures of practice bring Manitoba's *Kindergarten to Grade 8 Mathematics* and *Grades 9 to 12 Mathematics* Curriculum Framework of Outcomes to life with illustrations of various mathematics teaching and learning contexts in authentic classrooms. A collection of pictures of practice illustrates quality practices built from shared expertise.



"The teaching of mathematics is complex. It requires teachers to have a deep understanding of the mathematical content that they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grades. It also calls for teachers to be skilled at using instructional practices that are effective in developing mathematics learning for all students."

(NCTM, *Principles to Actions: Executive Summary* 4)

- **Understanding the Curriculum**
- **Embedding the Interrelated Mathematical Processes**
- **Applying Effective Teaching Practices**
- **Bridging Assessment and Instruction**



"A lot of scientific evidence suggests that the difference between those who succeed and those who don't is not the brains they were born with, but their approach to life, the messages they receive about their potential, and the opportunities they have to learn."

(Boaler 5)

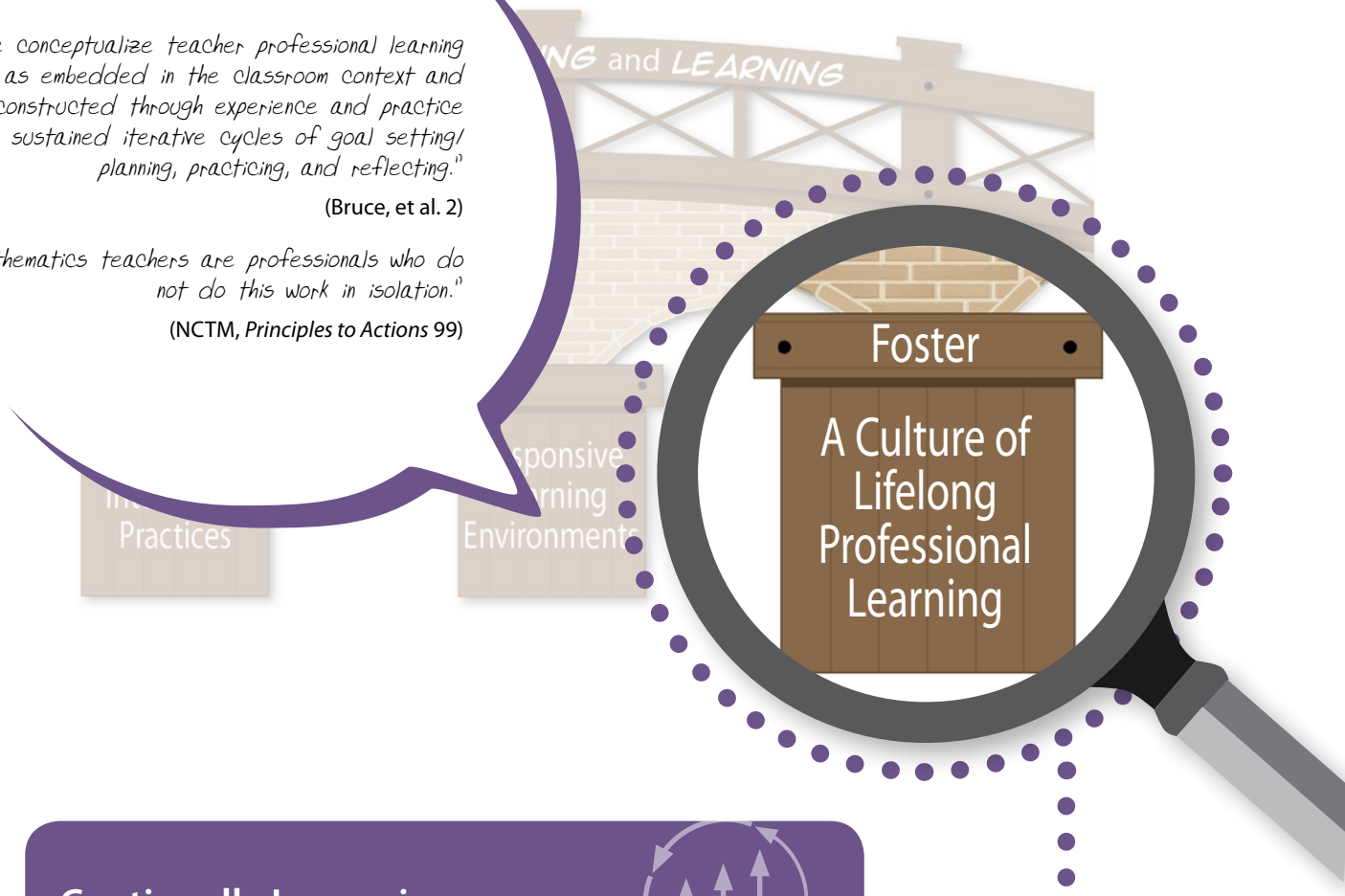
- **Ensuring Access and Equity**
- **Developing Mindsets**
- **Nurturing Learning Communities**

"We conceptualize teacher professional learning as embedded in the classroom context and constructed through experience and practice in sustained iterative cycles of goal setting/planning, practicing, and reflecting."

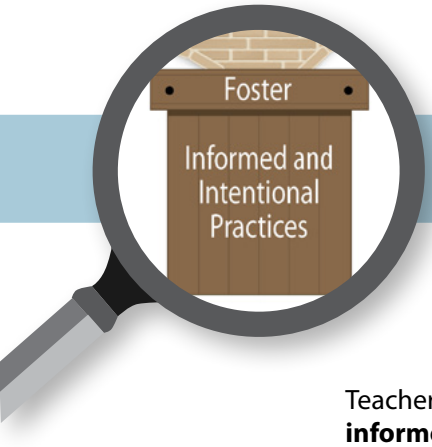
(Bruce, et al. 2)

"Mathematics teachers are professionals who do not do this work in isolation."

(NCTM, *Principles to Actions* 99)



- Continually Improving
- Collaborating
- Reflecting



Foster Informed and Intentional Practices

Teachers of mathematics need to have a clear understanding of the mathematics curriculum. The **informed and intentional practices** pillar encompasses four components that educators continually seek to address:

- **Understanding the Curriculum**
- **Embedding the Interrelated Mathematical Processes**
- **Applying Effective Teaching Practices**
- **Bridging Assessment and Instruction**

Grade-level learning outcomes cannot be treated as isolated skills and checklists of activities. The teaching and learning of mathematics includes posing problems that promote and connect conceptual understanding, problem solving, and reasoning. Additionally, it includes engagement with mathematics in which the goal of learning experiences focuses on students making sense of mathematics so they can learn and use it, both in and outside school.

Student learning of mathematics "depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum" (Ball and Forzani 17). It is essential that educators plan and develop the knowledge, resources, tools, and strategies to analyze student data to make informed and intentional instructional decisions. By gathering the best evidence about what students have learned, understanding mathematics, embedding the seven mathematical processes, and providing meaningful learning experiences in the classroom, teachers can bridge assessment and instruction to support students' mathematical thinking and learning.

Understanding the Curriculum

To plan purposeful instruction, teachers need to be knowledgeable about the curriculum.

Three areas to consider while planning and teaching mathematics are

- knowing and making sense of mathematics for teaching
- understanding learning progressions
- applying big ideas

Applying Effective Teaching Practices

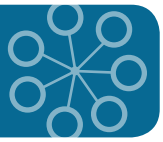
To enable students to become mathematical thinkers, teachers need effective actions and strategies.

The National Council of Teachers of Mathematics identifies a set of eight high-leverage teaching practices that promote deep learning of mathematics:

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

(Excerpted from *Principles to Actions* 10)

Embedding the Interrelated Mathematical Processes

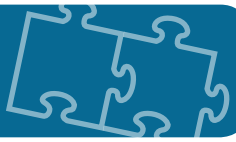


The mathematical processes allow students to engage in thinking about mathematics, and support the acquisition and use of mathematical knowledge and foundational skills that develop conceptual understanding.

The seven processes to be integrated within learning are

- Communication
- Connections
- Mental mathematics and estimation
- Problem solving
- Reasoning
- Technology
- Visualization

Bridging Assessment and Instruction



Assessment is embedded in the learning process and is closely connected with curriculum and instruction. Its role is to guide teaching and to support student learning.

Learning for all students is enhanced when assessment is designed with various purposes in mind:

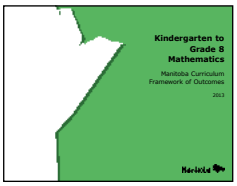
- assessment *for* learning
- assessment *as* learning
- assessment *of* learning

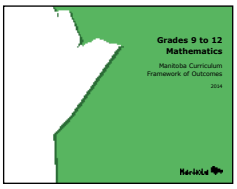
Understanding the Curriculum

The actions and decisions teachers make in mathematics classrooms need to nurture and deepen students' knowledge of mathematics. Educators need to be knowledgeable about the curriculum to plan and have students apply and make sense of mathematical concepts. While planning purposeful mathematics instruction, teachers focus on

- knowing and making sense of mathematics for teaching
- understanding learning progressions
- applying big ideas

As part of this planning, "both teachers and students need to be able to answer crucial questions:





- What mathematics is being learned?
- Why is it important?
- How does it relate to what has already been learned?
- Where are these mathematical ideas going?" (NCTM, *Principles to Actions* 13)

Understanding the mathematics behind curriculum outcomes can help develop teachers' mathematical proficiencies and guide their instructional decisions in the classroom.

Understanding mathematics is an ongoing process, and teaching it requires knowledge of how students learn and how they develop their mathematical thinking. Most students pass through predictable learning progressions or developmental stages or phases. Teachers who understand these progressions are able to make better instructional decisions.

The support document *Glance Across the Grades: Kindergarten to Grade 9 Mathematics* (Manitoba Education and Advanced Learning) allows educators to view the progression of learning outcomes across the grade levels. Educators can see how past learning connects to present and future learning.

Viewing learning outcomes as connections of big ideas allows students to think about the big picture of mathematics and its applications. Big ideas make connections between specific learning outcomes and mathematical concepts and provide purpose, focus, and meaning for teaching and learning the curriculum. *Glance Across the Grades* provides descriptions of big ideas within strands that can lead to making mathematics instruction more purposeful. Framing big ideas within mathematical content and learning experiences allows students to make essential connections between mathematical concepts and procedures.

Knowing and understanding the mathematics curriculum allows teachers to take advantage of every opportunity in students' learning, and helps them to facilitate connections of the mathematical ideas, symbols, and procedures to build conceptual understanding.

Questions to Guide Reflection and Discussion

- What does it mean to teach for understanding?
- How can we help students learn mathematics more thoroughly and deeply?
- What conditions could we put in place to promote deeper conceptual understanding?
- How can we structure lessons so that mathematical stages or phases are incorporated into the learning and all students experience success?
- How does all our professional learning in mathematics deepen our mathematical and curricular knowledge?
- What are the big ideas?
- How do big ideas guide instructional decisions?
- How do we use *Glance Across the Grades* as a planning tool?

Embedding the Interrelated Mathematical Processes

The following seven mathematical processes are intended to permeate teaching and learning:

- Communication
- Connections
- Mental mathematics and estimation
- Problem solving
- Reasoning
- Technology
- Visualization

The mathematical processes underlie and support mathematics content through all grades. In essence, they are the vehicles that allow students to engage in thinking about mathematics content. The processes are interwoven—the development of one can help with the development of others.

Planning for informed and intentional instruction includes framing content around the mathematical processes and the overarching goals of the curriculum: "The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills and their applications
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world" (Manitoba Education, *Kindergarten to Grade 8 Mathematics* 5).

Questions to Guide Reflection and Discussion

- How do we integrate the mathematical processes into our planning?
- How do we support students to make connections among topics?
- How do we provide opportunities for students to represent their understanding in different ways?
- How do we encourage students to communicate their understanding of mathematical ideas?
- What are some characteristics of classroom culture that support mathematical reasoning? How can we nurture this culture?
- What are the characteristics of a classroom culture that teaches through problem solving? How can we nurture this culture?

Applying Effective Teaching Practices

There is no "one right way" to teach mathematics; however, teachers do need to know and use a wide variety of instructional practices and strategies. Effective teaching has a big impact on student learning. Educators continually need to be aware of the impact they have on their students and, from the evidence of this impact, make decisions about adapting instruction.

NCTM identifies a core set of eight high-leverage teaching practices that promote deep learning of mathematics:

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking. (Excerpted from *Principles to Actions* 10)

These practices act as a common lens for supporting student learning. Teachers must know when and how to use these and other practices, evaluate the effect they have on student learning, and adjust accordingly.

Questions to Guide Reflection and Discussion

- What messages do we convey to students in our classroom practices?
- How do we want to see our mathematical practices evolve?
- How do we use research to inform our practices?
- What conditions could we put in place to promote deeper conceptual understanding?
- How can we provide learning opportunities where students are required to ask questions, think, and problem solve?
- What questions do we ask to promote reasoning, mathematical thinking, and sense making?

Bridging Assessment and Instruction

The role of assessment is to guide teaching and to support learning: "When learning is the goal, teachers and students collaborate and use ongoing assessment and pertinent feedback to move learning forward" (Manitoba Education, Citizenship and Youth, *Rethinking Classroom Assessment with Purpose in Mind* 5). Assessment of student progress includes gathering a body of evidence by listening to students, observing their actions, and analyzing their work (triangulation of data). Assessment *for, as*, and *of* learning all have a role in supporting student learning and must be used intentionally and purposefully.

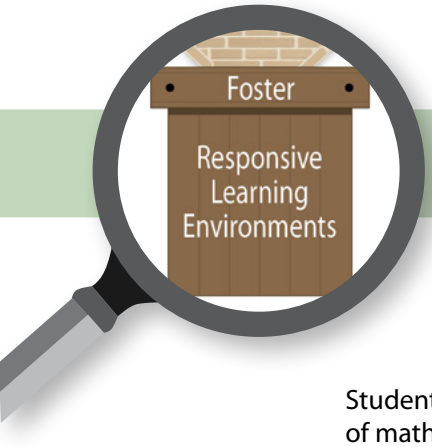
Actions that successfully unite and bridge assessment and instruction and make student thinking visible include

- gathering information to know students' learning preferences, interests, and curiosities
- assessing students' prior mathematical knowledge and experiences
- identifying intended learning outcomes and purposes for gathering evidence
- using assessment *for* learning to enhance students' learning and to provide descriptive feedback to move their learning forward
- making and modifying instructional decisions to facilitate students' learning so that mathematics makes sense to students
- using assessment *as* learning to develop and support students' metacognition
- using assessment *of* learning as a summative tool to confirm what students know and can do

The ultimate goal of assessment is to help students develop as independent, lifelong learners who regularly monitor and assess their own learning.

Questions to Guide Reflection and Discussion

- How would we describe our current assessment practices? What tools do we use?
- How can we incorporate more opportunities for students to reflect on their own mathematics learning?
- How could we improve our assessment approaches in order to use assessment to inform instructional decisions?
- How do we assess students' mathematical thinking?



Foster Responsive Learning Environments

Students and educators co-create safe and nurturing learning environments that support the learning of mathematics. To ensure equity, educators need to engage all students with the mathematical processes and enable all students to be successful in the thinking, learning, and doing of mathematics.

Teachers of mathematics continuously develop and adapt plans to create the conditions and opportunities necessary for each student to succeed. To support the success of all students, educators foster **responsive learning environments** that focus on the following three components:

- **Ensuring Access and Equity**
- **Developing Mindsets**
- **Nurturing Learning Communities**

Ensuring Access and Equity

To ensure mathematics is accessible and high-quality learning experiences are equitable, teachers consider

- the strengths and needs of each student
- high expectations for each student
- Indigenous perspectives
- cultural and contextual relevance
- differentiated supports

Developing Mindsets

In responsive learning environments, educators value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. As teachers and students work to develop positive mindsets, it is important to model and encourage

- an appreciation of mathematics
- a willingness to take risks
- curiosity and questioning
- productive struggle

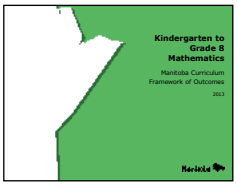
Nurturing Learning Communities

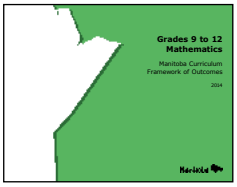
School leaders and teachers of mathematics establish and nurture the school and classroom environments. Successful learners of mathematics grow in physical, social, and emotional environments that value and nurture

- a sense of safety and belonging
- self-confidence and self-efficacy
- self-reflective learning
- student voice

Ensuring Access and Equity

Understanding the various strengths and needs of each student allows educators to support successful learning of mathematics with decisions that value and respect a diversity of student experiences and different ways of thinking. To ensure access to and equity in learning mathematics, educators need to provide all students with relevant, engaging, and high-quality learning experiences: "All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics" (NCTM, *Principles and Standards for School Mathematics* 12).





Integrating the seven interrelated mathematical processes into teaching and learning provides all students with access to and equity in the thinking, learning, and doing of mathematics.

By understanding the significance of Indigenous perspectives and the importance of cultural and contextual relevance teachers are better equipped to assess the strengths and needs of each learner. Establishing connections to the varying knowledge, life experiences, and backgrounds of individual learners is a key component in developing numeracy and sense making of mathematics.

Acknowledging that students differ in ways that significantly affect their learning, teachers need to provide differentiated supports that build on students' strengths and respond to students' needs.

Questions to Guide Reflection and Discussion

- How do we ensure high expectations for each individual learner in our learning community?
- What supports are we providing to ensure all students have opportunities to learn mathematics?
- How does the practice of embedding the seven mathematical processes ensure access to and equity in learning mathematics?
- What meaningful and productive learning experiences provide opportunities for sense making and developing thinking skills?
- What strategies beyond the incidental inclusion of topics and objects are we incorporating to ensure cultural and contextual relevance?
- What community resources provide contextual and cultural supports for mathematics teaching and learning?
- How can diversity of learners be used as a strength?
- What effective strategies for differentiation are we implementing?

Developing Mindsets

Responsive learning environments are most effective when teachers and students share opportunities to appreciate the power and beauty of mathematics and value the richness of diverse perspectives for problem solving and progressively developing mathematical ideas: "Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence" (Manitoba Education, *Kindergarten to Grade 8 Mathematics* 4).

Teachers who actively model a growth mindset create an environment where students openly look for and engage in finding a variety of strategies for solving problems. Students who are engaged in the productive struggle of doing and discussing mathematics explore alternatives, and develop as confident mathematical risk takers.

Questions to Guide Reflection and Discussion

- How can we model and practise an appreciation of mathematics?
- How are we practising mathematical discourse to support higher-level thinking?
- How is learning accomplished through problem solving?
- How is student thinking being made visible?
- What does productive struggle look like for students independently? cooperatively?
- What criteria do we use when selecting the challenging tasks that support meaningful learning?

Nurturing Learning Communities

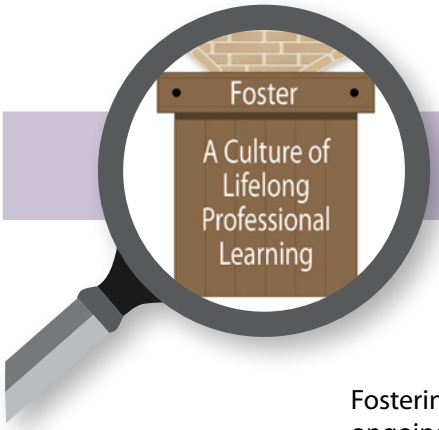
Students and educators work together to nurture relationships and co-create school and classroom environments to support mathematics learning. Intentional planning for these physical and social spaces increases the frequency and quality of opportunities for the thinking, learning, and doing of mathematics.

In a responsive learning environment, wellness is nurtured by creating a sense of safety and belonging for each learner; providing opportunities for all students to build self-confidence and self-efficacy; empowering individuals as self-reflective learners; and valuing each student's voice on the path to becoming an independent, lifelong learner.

In nurturing school and classroom environments, educators respond to the physical and social needs of students with decisions that build resilience, increase confidence, and improve students' abilities as mathematical thinkers.

Questions to Guide Reflection and Discussion

- How do we build relationships with students?
- How do our attitudes and beliefs about mathematics affect student learning, student attitudes, and how we teach?
- What strategies can we use to get to know the strengths and needs of each student?
- How are we using student and classroom profiles/data?
- What features in the physical environment of our school and classrooms create effective learning spaces?
- How is social-emotional learning supporting effective learning of mathematics?
- How are students engaged with setting and assessing personal goals?
- How are student voice and student choice actively valued in learning?
- How are students engaged as reflective learners?
- How do we engage students in a variety of ways of learning (e.g., cooperatively, independently)?
- What strategies can we use to support students in building self-confidence and self-efficacy?



Foster a Culture of Lifelong Professional Learning

Fostering **a culture of lifelong professional learning** requires professional educators to pursue ongoing professional growth, focusing on the following three components:

- Continually Improving
- Collaborating
- Reflecting

Continual improvement involves both self-reflection and collaboration and is informed by student work. To reach a goal of having student learning occur at the highest possible level, educators need to reflect on and continuously adjust their professional practice. Furthermore, collaboration among colleagues is required to learn from the experience of others: "In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics" (NCTM, *Principles to Actions* 5).

Continually Improving

As professionals, teachers of mathematics know their own learning is ongoing and they continually seek to improve and enhance their knowledge.

They need to attend to professional learning in four major areas:

- "Teachers' mathematical knowledge and their capacity to use it in practice
- Teachers' beliefs and dispositions that foster their continued learning
- Teachers' capacity to notice, analyze, and respond to students' thinking
- Teachers' collegial relationships and learning structures that can support and sustain their learning" (Doerr, Goldsmith, and Lewis, cited in NCTM, *Principles to Actions* 101).

Collaborating

Teachers embrace a culture of collaboration by sharing ideas, insights, and practices in order to improve both collectively and individually.

The collaborative roles could include

- teachers with students
- teacher colleagues within and across grade levels
- teachers with numeracy leaders and/or instructional coaches
- teachers with mathematics educators and mathematicians
- teachers with other stakeholders

Reflecting

Teachers continually reflect on their impact on students' learning and the subsequent actions taken.

To foster professional reflection, mathematics teachers

- focus on student thinking and student demonstrations of understanding
- seek opportunities to participate in formal and informal professional inquiry to improve their practice

Continually Improving

Educators work to increase their impact on students' mathematics learning and realize they need to be on a career-long path of professional growth. Teachers of mathematics can continually seek to improve their practice through cycles of professional learning, reading, dialogue, and reflection: "Learning occurs when we shift from professional certainty to conscious curiosity, from isolated individual to collaborative community member, and from passive technician to active researcher" (Lipton and Wellman 3).

The pillar of fostering **a culture of lifelong professional learning** focuses on the importance and components of professionalism and professional learning and overlaps with the other two pillars (fostering **informed and intentional practices** and **responsive learning environments**) in the content of the professional learning. In a quest for continual improvement and effective professional growth, teachers need to attend to professional learning in four major areas:

- "Teachers' mathematical knowledge and their capacity to use it in practice
- Teachers' beliefs and dispositions that foster their continued learning
- Teachers' capacity to notice, analyze, and respond to students' thinking
- Teachers' collegial relationships and learning structures that can support and sustain their learning" (Doerr, Goldsmith, and Lewis, cited in NCTM, *Principles to Actions* 101).

To grow in each of these four areas within the culture of lifelong professional learning, mathematics teachers need to embrace two key ideas: professional collaboration rather than isolation, and ongoing reflective practice. Collaboration and reflection are separated here to highlight their influence on continual improvement and ongoing professional growth. In reality, the three components of this pillar cannot be separated and are addressed simultaneously by educators.

Questions to Guide Reflection and Discussion

- What teacher beliefs and dispositions foster a journey of lifelong learning?
- What are some ways to create a culture of lifelong learning within the professional mathematics community?
- What process do we follow to reflect on and adjust our practice, with a goal to improve our students' learning?
- How do we measure the impact of professional learning?
- How do we determine when, what, and how to offer professional learning to teachers in our school division?

Collaborating

Teachers, as professionals, work collaboratively rather than in isolation. They embrace a culture of professional collaboration by sharing ideas, insights, and practices to improve both collectively and individually. The collaboration may involve work with teacher colleagues or work with mathematics coaches or specialists who serve as mentors. This collaboration focuses on teaching practice and refers to collaboration-to-learn rather than collaboration-to-decrease-workload (NCTM, *Principles to Actions* 99).

Collaboration can strengthen professional practice and should occur on many levels, including between educators in a school, across schools, across school divisions, and with Manitoba Education and Training.

The collaborative roles could include

- teachers and their students
- teacher colleagues who are open to collective observation, study, and improvement
- teachers across multiple grade levels who are open to sharing their pedagogical practices and knowledge
- teachers and numeracy leaders, divisional mathematics specialists, or instructional coaches
- teachers and mathematicians or mathematics educators
- teachers with other stakeholders (e.g., parents, Elders)

Effective collaboration takes time, so school leaders need to create and support opportunities for teachers to collaborate. School divisions and schools that value collaboration as a part of ongoing professional learning will find ways to include it in the school day and the school year.

Questions to Guide Reflection and Discussion

- How does collaborative professional learning improve our teaching practice and ultimately student learning?
- How pervasive is collaboration among teachers in our school division?
  - Within grade-level teams
  - With teachers at other grade levels
  - With staff at other schools (e.g., feeder)
- What roles help create a culture of collaboration?
  - What role does the teacher play?
  - What role does the numeracy leader play?
  - What role do the school leaders play?
  - What role does instructional coaching play?
- How committed are we to collaborating?
- What are some strategies to overcome barriers to making collaboration happen?

Reflecting

As professionals, teachers are willing to be reflective of their teaching practice. Reflection is a crucial part of the professional inquiry cycle: "Effective mathematics teaching results from purposeful planning. Highly effective teachers collaborate to design detailed mathematics lessons and then reflect on the effectiveness of those plans for student learning, in a cycle of continuous improvement" (NCTM, *Principles to Actions* 103). Continuous improvement requires teachers to engage in a cycle of intentional reflection on their impact on students' learning and the subsequent action taken to make improvements. Reflective and responsive teaching is based on a triangulation of data, is success oriented, and occurs both in the moment and long-term.

To foster a culture of professional reflection, mathematics teachers

- reflect, before a lesson, on what they are trying to accomplish and how they will recognize that it has happened
- listen to the student voice and respond to feedback
- focus on student thinking and student demonstrations of understanding
- reflect, after a lesson, on what was accomplished, what worked well, and what needs to change
- continue to reflect in an iterative, cyclical process
- seek opportunities to participate in formal or informal professional, collaborative inquiry with a goal to evaluate and improve their practice as evidenced by student demonstrations of understanding

Having a conversation with a numeracy leader and/or co-teacher can enhance the process of personal reflection as they act as a "mirror" for reflection. Furthermore, as teachers reflect and make a personal commitment to modify their practice, sharing plans with a colleague may help them to be accountable to their plans. Teachers will also benefit from dialogue with a colleague who has observed their actions with students in a classroom. Initially, teachers may feel vulnerable inviting colleagues, numeracy leaders, and coaches into their classrooms. They will begin to feel less vulnerable as they receive non-judgmental feedback on their practice, with a focus on student demonstrations of understanding.

Questions to Guide Reflection and Discussion

- In what ways are we engaged in professional reflection?
- What impact does our professional reflection have on our teaching practice and on students' learning?
- What roles help create a culture of professional reflection?
  - What role does the teacher play?
  - What role does the numeracy leader play?
  - What role do the school leaders play?
  - What role does instructional coaching play?
- What supports or structures are required to sustain school-wide or division-wide professional inquiry?

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